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An adaptive exponentially weighted moving average-type control chart to monitor the process mean[☆]

Amitava Mitra[a,*], Kang Bok Lee[a], Subhabrata Chakraborti[b]

[a] Department of Systems & Technology, Harbert College of Business, Auburn University, Auburn, AL, 36849-5266, United States
[b] Department of Information Systems, Statistics and Management Science, Culverhouse College of Commerce, The University of Alabama, Box 870226, Tuscaloosa, AL, 35487, United States

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ABSTRACT

Exponentially weighted moving average (EWMA) control charts are typically used for faster detection of shifts in the process mean, relative to a Shewhart control chart, when the degree of shift is small. Normal guidelines suggest using a small (large) value of the weighting constant, λ, for detecting smaller (larger) shifts in the process mean. Prior research has suggested that the choice of λ should depend on the observed data and have considered the use of a weighting constant, that varies and adapts as monitoring continues and new data are collected. One such adaptive control chart, called the AEWMA chart, utilizes a rather computationally complex scheme to determine the weighting constant λ and it requires knowledge of the size of the shift, to specify whether it is "small" or "large". A complex two-phase optimization scheme is then solved to yield "good solutions". As an alternative, we propose an adaptive EWMA-type control chart that does not require knowledge of the degree of the shift. Further, the computational scheme is easier and completed in one stage. The performance of the proposed chart is studied using simulations, where the degree of the shift in the process mean is varied over a wide range of values. Based on the average run length (ARL), as a performance measure, the proposed chart is demonstrated to perform uniformly better than the traditional EWMA chart with a constant weight.

The proposed chart also performs better than the AEWMA chart for moderate to large shifts.

1. Introduction

The traditional Shewhart control chart technique, introduced in 1924 by Walter Shewhart, has been used in a variety of manufacturing and service industries. The commonly used control charts are those based on the sample mean ($\bar{X}$), the range (R), the standard deviation (s), individual data (X) and the moving range (MR). In the first two situations ($\bar{X}$ and R, $\bar{X}$ and s), it is assumed that independent and identically distributed observations (X) are selected from a process, in subgroups, where the subgroup size is denoted by n. In the third situation, the subgroup size is 1 and the process is monitored based on individual observations (Mitra, 2016).

The popular Shewhart control charts, while easy to implement, have the disadvantage that for small shifts in the process parameters, for example, the mean, it takes a rather long time to detect the shift. Consequently, control charts have been developed with the purpose of faster detection of small changes in the process parameters. These include the cumulative sum (CUSUM) chart and the exponentially-weighted moving average (EWMA) control chart (Albin, Kang, & Shea, 1997; Chen & Elsayed, 2002; Domangue & Patch, 1991; Klein, 1996; Lucas & Saccucci, 1990; Roberts, 1959). The traditional EWMA control chart is based on the monitoring statistic

$$Z_i = \lambda X_i + (1 - \lambda) Z_{i-1} \tag{1.1}$$

where the starting exponentially-weighted smoothed value, Z_0, is often chosen to be the target value of the mean (or the first observed value) and $0 < \lambda \leq 1$ is the smoothing constant. Note that when $\lambda = 1$, the EWMA statistic reduces to the monitoring statistic of a Shewhart chart for individual data. When the monitoring statistic is based on the sample means, a similar comment applies. An alternative form of the EWMA statistic for individual data is given by

$$Z_i = \lambda \sum_{j=0}^{i-1} (1 - \lambda)^j X_{i-j} + (1 - \lambda)^i Z_0 \tag{1.2}$$

which indicates that while monitoring the data at the current period i, the weights assigned to the prior observations (X_{i-j}) decline

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* Corresponding author.
E-mail addresses: mitraam@auburn.edu (A. Mitra), kbl0009@auburn.edu (K.B. Lee), schakrab@cba.ua.edu (S. Chakraborti).

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exponentially, as in a geometric series, the further removed the observation is from the current period. When the process mean μ is specified or known, say μ_0, the upper and lower control limits, denoted by UCL and LCL, respectively, are chosen based on a selected level of a false alarm rate (Type I error). Another representation of the EWMA statistic is given by

$$Z_i = Z_{i-1} + \lambda e_i \quad (1.3)$$

where $e_i = X_i - Z_{i-1}$, is the residual at time i, which represents the deviation of the observed value at the current time period from the smoothed or the predicted value at the previous time period (a forecast for the current period). Prior researchers (Chen & Elsayed, 2002; Jones, 2002; Champ; Lucas & Saccucci, 1990) have studied the properties and performance of the EWMA chart. They noted that the EWMA chart overcomes the drawback of the traditional Shewhart chart for small shifts in the process parameter, i.e., the process mean. In particular, for an EWMA chart, the average run length (ARL) for the detection of a shift is usually smaller than that for a Shewhart chart. These studies have demonstrated that for "small" shifts in the process mean, a small value of λ is desirable, while for "large" shifts in the process mean, a large value of λ is desirable. As the degree of the shift becomes large, the optimal value of λ approaches 1, implying that the EWMA chart reduces to the Shewhart chart for the individuals data. When the observations, X_i, are independent and identically distributed from a $N(\mu_0, \sigma_0^2)$ process, where μ_0 and σ_0^2 are known or specified, the steady state EWMA control limits are given by

$$(\text{UCL}, \text{LCL}) = \mu_0 + L\sqrt{\frac{\lambda}{(2-\lambda)}}\ \sigma_0 \quad (1.4)$$

where L is a design parameter (charting constant) based on a chosen false alarm rate. Since the original EWMA chart uses a fixed value of the smoothing constant, λ, it does not do well, uniformly, over the entire range of possible shifts in the process mean. For example, it is known that the EWMA chart with a small λ does not detect "large" shifts as efficiently as the Shewhart chart.

Lucas and Saccucci (1990) analyzed the performance of the EWMA chart utilizing a Markov Chain approach. Using simulations, they demonstrated that for small shifts in the process mean, the EWMA chart has better performance compared to the Shewhart chart, when the average run length (ARL) is used as a performance criterion. They also investigated the performance of a combined Shewhart and EWMA scheme in order to provide protection for small and large shifts in the process mean.

Since the EWMA statistic also appears as the optimal forecasting method for some time series model and is easier to apply than the CUSUM chart, there is a lot of interest and consequently a substantial literature on EWMA charts. Klein (1996) obtained ARL profiles for both the composite Shewhart-EWMA scheme and the Shewhart chart with specified run rules. The ARL profiles were shown as a function of the magnitude of the shift in the process mean. The composite scheme using a Shewhart – EWMA combination depicted a better profile. Albin, Kang, and Shea (1997) studied several schemes using individual observations. They investigated combinations of the moving range, EWMA, and Shewhart charts with and without runs rules. Using a simulation approach, they studied the ARL for shifts in the process mean as well as the process standard deviation. Jiang, Shu and Apley (2008) studied the performance of some adaptive CUSUM procedures using EWMA-based shift estimators. Using the concept of a loss function, Wu, Wang and Wang (2008) proposed an adaptive control chart to monitor the process mean and variance. Wu, Yang, Khoo and Yu (2010) developed an optimization algorithm to design CUSUM charts to monitor both the process mean and variability. They demonstrated that the performance of their control chart is nearly as good as an optimal scheme that uses three individual CUSUM charts. Prior research has also been conducted on integrating statistical process control (SPC) techniques with engineering process control (EPC) methods that use run-to-run controllers to dynamically control a process. For example, Fan and Lin (2007) used an adaptive dual-response optimizing controller that serves as a recipe regulator between consecutive runs in wafer fabrication for semiconductor manufacturing processes. Our focus, however, is not on this aspect of dynamic engineering process control.

Some researchers have recently considered data driven control charts by using chart design parameters (including the charting constant) that adapt and adjust (upward or downward) dynamically as the data are sequentially collected and observed. This is an interesting and practically appealing idea. To this end, another approach for faster detection of small and moderate shifts in the process mean is that of using "variable parameters" control charts. Here, the design parameters such as the sample size, sampling interval, and the charting constant are chosen to determine the location of the control limits that are chosen adaptively (Reynolds & Arnold, 2001).

These design parameters may take on different values based on the most recent information from the process. Tagaras (1998) provides a review of adaptive control charts where the design parameters change dynamically. De Magalhães and Moura Neto (2005) developed a variables parameter $\bar{X}$ and R charts so as to minimize the expected cost. He and Grigoryan (2006) considered joint monitoring of the process through $\bar{X}$ and R charts where they utilize two-stage sampling through double sampling and variable sample sizes. In addressing the issue of non-normality of the observations, through a variables parameter $\bar{X}$ chart, Lin and Chou (2007) considered mean shifts in processes. In the context of multivariate processes, a variables plan that considers changing the sample size, sampling interval, and the action limit through an adaptive procedure for a Hotelling's T^2 chart was investigated by Chen (2007). This study was later extended by Chen and Hsieh (2007) where the sampling interval was kept constant but the sample size and control limit varied adaptively. Our focus, in this paper, is not on a variables sampling plan where the sample size, sampling interval, or control limits change from sample to sample. On the contrary, our objective is to keep the monitoring process simple. Through the choice of adaptive weights, based on the observations, our goal is to facilitate detection of a shift in the process mean.

Hubele and Chang (1990) developed two adaptive EWMA control schemes. The weighting constant is updated using a Kalman filter algorithm. Shu (2008) developed an adaptive EWMA chart for monitoring process variances by using a Markov chain model. Steiner and Jones (2009) considered risk-adjusted survival time monitoring with an updating EWMA control chart. Mahmoud and Zahran (2010) considered a multivariate adaptive EWMA control chart that was a combination of the multivariate EWMA (MEWMA) chart and a Shewart χ^2 -chart. Zhang, Li and Wang (2010) investigated the simultaneous monitoring of the process mean and variability through a multivariate control chart. Saleh, Mahmoud and Abdel-Salam (2012) studied the performance of the adaptive EWMA control chart with estimated parameters using a Markov chain approach. Huang, Shu and Su (2014) conducted an evaluation of adaptive EWMA schemes. While the Markov chain method was originally used to approximate ARL performance, it may have slow convergence. To overcome this, they extend the piecewise collocation method and the Clenshaw-Curtis (CC) quadrature method. Aly, Hamed and Mahmoud (2015) considered he design of an adaptive EWMA control chart by considering the process mean shift as a random variable with a certain probability distribution. Aly, Mahmoud and Hamed (2015)) studied the performance of the multivariate adaptive EWMA chart with estimated parameters. They considered the practitioner-to-practitioner variation by using different

Phase I samples by different practitioners. Aly, Saleh, Mahmoud and Woodall (2015) further investigated the performance of the adaptive EWMA chart with estimated parameters. Rather than utilize the expected value of the in-control ARL, they used the standard deviation of the ARL metric to study the performance. Tang, Castagliola, Sun and Hu (2017) considered a variable sampling interval adaptive EWMA chart to monitor shifts in process mean.

2. Adaptive EWMA chart

To capture the desirable features of both the EWMA and the Shewhart charts, that is to be able to detect both large and small shifts effectively, that is data driven, Capizzi and Masarotto (2003) proposed an adaptive EWMA (AEWMA) chart where the smoothing constant λ changes based on the magnitude of the residual at time *t*. Thus, the AEWMA chart examines what has happened with the data in the past, adapts and uses a possibly new weighting constant, as it monitors the current observation. Their monitoring statistic is given by

$$Z_t = Z_{t-1} + \phi(e_t), \; t = 1, 2, \ldots \tag{2.1}$$

where $Z_0 = \mu_0$ and $e_t = X_t - Z_{t-1}$. The choice of the score function, $\phi(e_t)$, defines the adaptive nature of the statistic. The score function is proposed to be odd and monotone increasing in e_t, and when $|e_t|$ is small, it should be close to λe_t, while when $|e_t|$ is large, it should be close to e_t. They proposed three score functions given as follows:

$$\phi_{hu}(e) = \begin{cases} e + (1-\lambda)k & \text{if } e < -k \\ \lambda e & \text{if } |e| \le k \\ e - (1-\lambda)k & \text{if } e > k \end{cases} \tag{2.2}$$

$$\phi_{bs}(e) = \begin{cases} e\left\{1 - (1-\lambda)(1-e/k)^2\right\} & \text{if } |e| \le k \\ e & \text{otherwise} \end{cases} \tag{2.3}$$

and

$$\phi_{cb}(e) = \begin{cases} e & \text{if } e \le -p_1 \\ -\tilde{\phi}_{cb}(1-e) & \text{if } -p_1 < e < -p_0 \\ \lambda e & \text{if } |e| \le p_0 \\ \tilde{\phi}_{cb}(e) & \text{if } p_0 < e < p_1 \\ e & \text{if } e \ge p_1 \end{cases} \tag{2.4}$$

where $0 < \lambda \le 1$, $k \ge 0$, and $0 \le p_0 < p_1$ denote suitable constants and $\tilde{\phi}_{cb}(\bullet)$ is the cubic polynomial that makes $\phi_{cb}(\bullet)$ and the first derivative continuous, i.e.,

$$\tilde{\phi}_{cb}(e) = \lambda e + (1-\lambda)\left(\frac{e-p_0}{p_1-p_0}\right)^2 \times\left\{2p_1 + p_0 - (p_0+p_1)\left(\frac{e-p_0}{p_1-p_0}\right)\right\} \tag{2.5}$$

The first two ϕ-functions were based on Huber's function (Huber, 1981) and Tukey's bi-square function (Beaton and Tukey, 1974). The third ϕ-function blends the ϕ-functions of the Shewhart and EWMA charting schemes.

In the above ϕ-functions, λ and *k* are the design parameters associated with the control chart. Further, the chart signals when the monitoring statistic Z_t falls outside some bounds $\mu_0 \pm h\,\sigma_0$, where σ_0 is the process standard deviation and the charting constant *h* is calculated for a chosen false alarm rate (level of a Type I error). The monitoring statistic Z_t given by Eqn. (2.1) can be re-expressed as

$$Z_t = w(e_t)X_t + [1 - w(e_t)]Z_{t-1} \tag{2.6}$$

where $w(e) = \phi(e)/e$, which represents the adaptive weight associated with the adaptive EWMA (AEWMA) monitoring statistic of Capizzi and Masarotto (2003).

Note that while the idea is appealing, there are some difficulties in obtaining the "optimal" values for the design parameters λ, *k*, and *h* for the implementation of the AEWMA chart. The usual concept for determining optimal design parameter values is to minimize the out-of-control ARL, for a specified shift in the process mean, while maintaining a desired in-control ARL(ARL_0) . The difficulty with this approach is that, as demonstrated also by Lucas and Saccucci (1990), tuning the chart, that is finding the "optimal" values, depend on the magnitude of the shift, which is generally unknown. Further, since the optimal value of λ varies quite a bit based on the size of the shift, the ARL of a control chart designed for a small shift may be quite different from that designed for a large shift.

In this context, finding the optimal values of the design parameters $\theta = (\lambda, k, h)$ computationally, as suggested by Capizzi and Masarotto (2003), is quite complex. Their proposed strategy requires the selection of a "small shift", say denoted by μ_1, and the selection of a "large shift", as denoted by μ_2, by the user. Their computational procedure is a two-phased approach. In the first phase, the optimal parameter, θ, is found based on a solution to the following optimization problem:

$$\begin{aligned} &\underset{\theta}{\text{Minimize}} \quad \text{ARL}(\mu_2, \theta) \\ &\text{Subject to : ARL}(0, \theta) = B \end{aligned} \tag{2.7}$$

where ARL (0, θ) denotes the desired in-control ARL corresponding to a chosen false alarm rate.

The second phase of their computational approach involves first choosing a small positive constant, α, say $\alpha = 0.05$. The "optimal" θ is found as the solution to the problem:

$$\begin{aligned} &\underset{\theta}{\text{Minimize}} \quad \text{ARL}(\mu_1, \theta) \\ &\text{Subject to : ARL}(0, \theta) = B \\ &\quad \text{and ARL}(\mu_2, \theta) < (1+\alpha)ARL(\mu_2, \theta^*) \end{aligned} \tag{2.8}$$

The idea is to find the scheme with the minimum ARL when the mean is at μ_1, among those for which ARL at μ_2 is "nearly minimum". Thus, knowledge (or assumption) of μ_1 and μ_2 is necessary in order to implement the complex two-phased optimization problem. In practical applications, such an approach is difficult to implement since the degree of shift may not be known.

Thus, our motivation is to propose an adaptive EWMA-type control chart for which the design parameters can be found more easily so that its application can be facilitated in practice. As opposed to the scheme proposed by Capizzi and Masarotto (2003), where three design parameters are to be determined, we propose an adaptive EWMA-type chart, in which the smoothing constant is chosen adaptively based on the data and is the only parameter to be estimated. Further, no assumption needs to be made on the size of a shift, whether it is "small" or "large", in the process mean, making it more realistic in terms of application. This is described in the next section.

3. Proposed adaptive control chart

In the proposed approach, we incorporate and use a "look-back" feature, as defined by a parameter, $r(\ge 2)$, in the definition of the EWMA-type monitoring statistic Thus, rather than considering the entire history involving all the past observations, we consider, starting with the current observation, using the previous *r*-1 observations. Hence, we keep the spirit of the idea behind the original EWMA statistic, that is including the past observations to monitor the present, but we only choose to weigh the last (*r*-1) observations, with the weights declining in a geometric series as in the original EWMA. The motivation behind using the look-back parameter is to overcome the inertial effect of the distant past observations in detecting a shift in the process mean by the EWMA chart

(Woodall & Mahmood, 2005; Yashchin, 1995). In our proposal, the weights assigned to the current observation, and the (*r*-1) prior observations are determined adaptively. Observations that are further back in the past from the current period *i*, will receive less weight, with the weights decreasing exponentially, while observations that are more distant than the *r*th, (*r*+1)th, and further from the current period will receive a zero weight. As we will see, the weights will be influenced by the magnitude of the deviation of the observations from the specified target value of the mean, μ_0.

At time *i*, for a given *r*, we define the statistic:

$$M(i, r, w) = \sum_{j=i-r+1}^{i} w_i (1 - w_i)^{i-j} \left[X_j - \mu_0\right]^2 \tag{3.1}$$

The "optimal" adaptive weight, for given values of *i* and *r*, is defined as

$$w^*(i, r) = \arg\max_w M(i, r, w), \ w \in [0, 1] \tag{3.2}$$

Thus, for a given value of *r*, at a certain time period *i*, the value of *w* that maximizes $M(i, r, w)$ is the proposed adaptive weight, denoted by $w^*(i, r)$, to be found from Eq. (3.2). This optimal weight depends on the observations, starting with the current period and looking back (*r* – 1) periods. Using this optimal weight, the monitoring statistic for the proposed adaptive EWMA-type chart is

$$M(i, r, w^*) = \sum_{j=i-r+1}^{i} w^*(i, r)\,(1 - w^*(i, r))^{i-j} * \left[X_j - \mu_0\right]^2 \tag{3.3}$$

The adaptive EWMA-type control chart that uses the monitoring statistic given by Eq. (3.3) is labeled as the AE chart. The computation and use of the proposed adaptive chart is rather simple relative to the AEWMA chart of Capizzi and Masarotto (2003). First, our proposal does not require values for the size of a "small" and that of a "large" shift. Further, computationally it does not require a two-stage optimization procedure, for which the determination of the chart design parameters are quite involved. The simplicity of the proposed chart, in practice, is seen as a major advantage. However, note that the AE chart does require a user-specified value of the look-back parameter *r*, which is a measure of the degree of emphasis on prior observations as selected by the user.

3.1. Insights on the proposed adaptive chart (AE)

To obtain the optimal value of the adaptive weight $w^*(i, r)$, for a given value of *i* and *r*, we maximize the value of $M(i, r, w)$ as given by Eq. (3.1). Using $\mu_0 = 0$, it is observed that, for $i = 2$, and $r = 2$, for example,

$$\begin{aligned} M(2, 2, w) &= \sum_{j=1}^{2} w_2 (1 - w_2)^{2-j} X_j^2 \\ &= w_2 (1 - w_2) X_1^2 + w_2^2 X_2^2 \end{aligned} \tag{3.4}$$

Maximizing $M(2, 2, w)$ with respect to *w*, we find

$w^*(2, 2) = (X_2^2 + X_1^2)/2X_1^2$.

Hence, since $w^* \in (0, 1)$, we have

$w^*(2, 2) = min\,[(X_2^2 + X_1^2)/2\,X_1^2, 1]$.

By induction, it can be shown that for $r = 2$, for any observation at period *i*, the optimal adaptive weight $w^*(i, 2)$ is given by

$$w^*(i, 2) = \min\left[(X_i^2 + X_{i-1}^2)/2X_{i-1}^2, 1\right] \tag{3.5}$$

Hence, it can easily be shown that the monitoring statistic, $M(i, 2, w^*)$, for the proposed adaptive EWMA-type chart when $r = 2$ is

$$M(i,\ 2,\ w^*) = \begin{cases} w^*(i, 2)(1 - w^*(i, 2)X_{i-1}^2 + w^*(i, 2)X_i^2, & if\ 0 < w^*(i, 2) < 1 \\ X_i^2, & if\ w^*(i, 2) = 1 \end{cases} \tag{3.6}$$

The simulation study reported in the next section demonstrates the performance of the various control charts. Here 30 observations are generated from an in-control process and starting with period 31, 30 more observations are generated from the process with a shifted mean. So, if we are currently in period $i = 35$, and $r = 35$, we have 5 observations from the shifted process and 30 observations from the in-control process to determine the adaptive weight $w^*(i, r)$.

4. Performance of the proposed adaptive control chart

A simulation study is conducted to examine the performance of the proposed adaptive control chart (AE). For a chosen magnitude of a shift in the process mean, the selected performance measure is the average time to first detection of the shift (the average run length, ARL). Initially, the process mean is assumed to be in a state of control with mean $\mu_0 = 0$ and variance $\sigma_0^2 = 1$. Hence, the observations prior to the shift are generated from N(0,1). The degree of shift in the process mean is represented by $\delta\sigma_0$, where δ is a chosen parameter that is selected to vary between 0.1 and 4.0. Thus, "small" and "large" degrees of shift of the process mean in the simulation are represented by values of δ. Using 10,000 replications, the run length, the mean run length (ARL) and the standard error of the run length (SERL) are calculated for the proposed adaptive control chart. The SERL is the standard deviation of the run length (SDRL) divided by the square root of 10,000, the number of replications.

For an in-control process, three values of α, the Type I error probability (the false alarm rate) were selected, namely 0.05, 0.01, and 0.005. For each selected value of α, the bound *h* for the detection of a shift was determined through an empirical approach. For a selected value of *r*, the look-back parameter, for each value of the current time period *i*, the monitoring statistic $M(i, r, w^*)$ is found over 10,000 replications, when the observations are from an in-control process. The values of the statistic are ranked and the upper $100\alpha^{th}$ percentile, for example, the 500th largest value if α is chosen to be 0.05, is selected as the bound, *h*. This bound is then used in the second phase of the simulation when a change occurs in the process mean. For the initial set of data from the in-control process, 30 observations are generated from a N(0, 1) distribution. Subsequently, for the shifted process, the process mean value, μ_1, was chosen to be as follows: 0.10, 0.25, 0.50, 0.75, 1.0, 1.5, 2.0, 2.5, 3.0, and 4.0, respectively. The selected number of look-back periods, *r*, starting with the current observation, was chosen to be $r = 2$, 3, 5, and 10, respectively.

Table 1 shows the ARL and the SERL of the proposed adaptive control chart (AE) for $\alpha = 0.05$ over 10,000 replications. For example, for a very small shift in the process mean of 0.1 standard deviations, when $r = 2$, the out-of-control (OOC) average run length (ARL) is 22.837, with a standard error of 0.098. As *r* increases to 3, 5, or 10, the ARL increases to 24.738, 26.560, and 27.100, respectively. These results represent the inertial effect on shift detection. To detect a smaller shift in the process mean, a "look-back" feature where *r* is small, say 2 or 3, will detect the shift faster than when *r* is large, say 5 or 10. As the degree of shift in the process mean increases, as expected, in general the OOC ARL decreases. For shifts as large as 3 standard deviations, note that for a small *r*, such as $r = 2$, the ARL for detection is 1.368, with a standard error of 0.006. Note that not only is the OOC ARL smaller, but also the SE,

Table 1
Mean (Standard error) of run length of adaptive control chart ($\alpha = 0.05$, $\mu_0 = 0$, $\sigma = 1$).

AE				
μ_1, σ	$r = 2$	$r = 3$	$r = 5$	$r = 10$
0.1, 1	22.837	24.738	26.560	27.100
	(0.098)	(0.089)	(0.076)	(0.071)
0.25, 1	22.675	23.798	25.608	26.700
	(0.098)	(0.094)	(0.083)	(0.039)
0.40, 1	20.566	22.436	25.124	25.974
	(0.104)	(0.099)	(0.086)	(0.081)
0.50, 1	19.757	21.044	23.477	24.959
	(0.104)	(0.103)	(0.095)	(0.087)
0.75, 1	15.577	17.516	19.807	21.390
	(0.104)	(0.106)	(0.105)	(0.102)
1.0, 1	11.521	12.770	15.282	17.280
	(0.091)	(0.095)	(0.102)	(0.105)
1.5, 1	5.357	6.234	7.209	8.807
	(0.049)	(0.056)	(0.064)	(0.075)
2.0, 1	2.915	3.186	3.593	4.379
	(0.024)	(0.027)	(0.031)	(0.038)
3.0, 1	1.368)	1.328	1.456	1.639
	(0.006)	(0.006)	(0.008)	(0.010)
4.0, 1	1.054	1.031	1.065	1.111
	(0.002)	(0.002)	(0.003)	(0.004)

Table 2
Mean (Standard Error) of run length of EWMA Chart ($\alpha = 0.05$, $\mu_0 = 0$, $\sigma = 1$).

EWMA					
μ_1, σ	$w = 0.1$	$w = 0.3$	$w = 0.5$	$w = 0.7$	$w = 0.99$
0.1, 1	30	30	29.997	29.913	29.256
	(0.000)	(0.000)	(0.001)	(0.013)	(0.038)
0.25, 1	30	30	29.995	29.86	29.064
	(0.000)	(0.000)	(0.003)	(0.016)	(0.042)
0.40, 1	30	29.999	29.979	29.771	28.642
	(0.000)	(0.000)	(0.005)	(0.021)	(0.049)
0.50, 1	30	29.998	29.965	29.648	28.244
	(0.000)	(0.002)	(0.008)	(0.025)	(0.056)
0.75, 1	30	30	29.887	29.251	26.717
	(0.000)	(0.000)	(0.015)	(0.037)	(0.074)
1.0, 1	30	29.992	29.678	28.315	24.329
	(0.000)	(0.003)	(0.024)	(0.054)	(0.091)
1.5, 1	30	29.847	27.584	23.058	15.830
	(0.000)	(0.0015)	(0.064)	(0.096)	(0.103)
2.0, 1	29.999	27.332	19.480	12.940	7.767
	(0.000)	(0.063)	(0.102)	(0.094)	(0.068)
3.0, 1	22.754	7.7354	4.481	3.127	2.212
	(0.064)	(0.046)	(0.031)	(0.023)	(0.016)
4.0, 1	8.435	3.035	1.949	1.503	1.239
	(0.022)	(0.012)	(0.009)	(0.007)	(0.005)

for smaller values of r. For this same degree of shift, if r increases to 10, the OOC ARL is 1.639, with a standard error of 0.010. For larger shifts in the process mean, the inertial effect on shift detection diminishes. As an example, for a shift of 4 standard deviations, there is not much of a difference in the ARLs as r varies from 2 to 10. To summarize the results from this table, a small value of the "look-back" parameter r, will detect smaller shifts in the process mean faster, as measured by ARL, with also a reduced variability, as measured by SERL. This is because of the less inertial effect of observations from the in-control process.

Table 2 displays the ARL and SERL values of the traditional EWMA control chart for a given smoothing constant, w. The values of the smoothing constant are chosen to be 0.1, 0.3, 0.5, 0.7, and 0.99, respectively. Note that when $w = 0.99$, the EWMA statistic approaches the traditional Shewhart statistic for individuals. The parameters of the simulation are similar to that used for Table 1, with an $\alpha = 0.05$. Here, we do not have a look-back feature since the EWMA control chart considers all observations, prior to, and including the current observation. In comparing the performance of the AE control chart with the EWMA chart, it is seen that the proposed AE chart performs uniformly better than the EWMA chart over the range of the simulated parameters. First, when a value for OOC ARL (SERL) is shown as 30 (0.000), it means that for each of the 10,000 simulations, the EWMA control chart failed to detect the mean shift within the span of thirty new observations from the out-of-control process. So, in a sense, the OOC ARL is actually 30 or more. However, in our simulation scheme, since we considered up to a maximum of 30 new observations, we conclude that the OOC ARL is at least 30.

Next, we note that in comparing the results in Tables 1 and 2, over all degrees of shift in the process mean and all values of the weight (w) used in the EWMA chart, the AE chart has better performance than that of the EMWA chart. For example, for a shift in the process mean of one standard deviation, for the AE estimator with $r = 2$, the OOC ARL (SERL) is 11.521 (0.091). This dominates the OOC ARL of the EWMA chart for any level of the constant weight (w), the best scenario being for $w = 0.99$, when the corresponding values are 24.329 (0.091). If a one-tailed t- test for comparison of ARLs were to be performed, the t-statistic is −99.52 and the p-value is found to be < 0.0001. For any reasonable level of a chosen level of significance, we conclude that the ARL for the AE control chart is less than the corresponding ARL for the best EWMA chart.

If a similar comparison were to be made for a shift in the process mean of two standard deviations the corresponding values of the OOC ARL (SERL) for the AE estimator with $r = 2$ are 2.915 (0.024), while for the traditional EWMA with a constant weight in the best scenario with a $w = 0.99$ are 7.767 (0.068). As before, if a one-tailed t – test for comparison of ARLs were to be performed the t-statistic is −67.29 and the p-value is found to be < 0.0001. As the degree of shift in the process mean increases, the performance of the proposed AE control chart, relative to the EWMA chart, diminishes as expected. For large shifts in the process mean, the OOC ARL should decrease for all control charts and so the relative advantages of the AE chart is reduced. One can infer that for the AE chart, which finds an "optimal" adaptive weight based on the data, and uses the depth of the "look-back" feature through the choice of a parameter, the overall performance dominates the performance of the chart that uses a constant weight.

Table 3 shows the ARL and SERL of the AE control chart with a chosen $\alpha = 0.01$ over 10,000 replications. As in the previous simulations, the "look-back feature" as indicated by the parameter r is varied. The performance is similar to that observed in Table 1. Obviously, with a reduced value in the choice of α, the ARLs in detecting a shift are increased, relatively speaking.

Table 4 shows the ARL and SERL of the EWMA control chart with a chosen $\alpha = 0.01$ over 10,000 replications. In comparing the performance of the AE control chart to the EWMA control chart, the proposed chart is found to perform uniformly better, as we found previously in the case of $\alpha = 0.05$. As an example, for a shift in the process mean of one standard deviation, for the AE estimator with $r = 2$, the OOC ARL (SERL) is 20.851 (0.103). This dominates the OOC ARL of the EWMA chart, for any level of the constant weight w, the best scenario being for $w = 0.99$, when the corresponding values are 28.085 (0.058). If a one-tailed t-test for comparisons of the ARLs were to be conducted, the t-statistic is −61.20 and the p-value is found to be < 0.0001. This demonstrates that the OOC ARL for the AE chart is less than the corresponding OOC ARL for the best EWMA chart under the situation, establishing the superiority of the AE chart. Such dominance is observed across all levels of the shift in the process mean.

As expected, as the chosen level of significance (α) decreases, the degree of superiority of the AE chart over the EWMA chart

Table 3
Mean (Standard Error) of run length of adaptive control chart ($\alpha = 0.01$, $\mu_0 = 0$, $\sigma = 1$).

AE				
μ_1, σ	$r=2$	$r=3$	$r=5$	$r=10$
0.1, 1	28.859	28.797	29.459	29.545
	(0.045)	(0.046)	(0.032)	(0.029)
0.25, 1	28.073	28.818	29.125	29.186
	(0.058)	(0.047)	(0.041)	(0.039)
0.40, 1	27.434	28.006	28.615	29.003
	(0.067)	(0.059)	(0.051)	(0.043)
0.50, 1	26.638	27.251	28.116	28.500
	(0.075)	(0.069)	(0.058)	(0.052)
0.60, 1	26.306	26.985	27.691	28.122
	(0.078)	(0.072)	(0.064)	(0.058)
0.75, 1	24.271	25.838	26.858	27.788
	(0.091)	(0.082)	(0.073)	(0.062)
1.0, 1	20.851	22.433	23.928	25.062
	(0.103)	(0.099)	(0.093)	(0.087)
1.5, 1	11.908	13.222	15.870	16.746
	(0.093)	(0.098)	(0.105)	(0.106)
2.0, 1	5.640	6.377	7.598	8.686
	(0.051)	(0.057)	(0.067)	(0.075)
3.0, 1	1.919	1.983	2.309	2.547
	(0.013)	(0.014)	(0.017)	(0.020)
4.0, 1	1.172	1.137	1.201	1.304
	(0.005)	(0.004)	(0.005)	(0.006)

Table 4
Mean (Standard Error) of Run Length of EWMA Chart ($\alpha = 0.01$, μ_0, $\sigma = 1$).

EWMA					
μ_1, σ	$w=0.1$	$w=0.3$	$w=0.5$	$w=0.7$	$w=0.99$
0.1, 1	30	30	30	29.989	29.886
	(0.000)	(0.000)	(0.000)	(0.004)	(0.013)
0.25, 1	30	30	30	29.987	29.791
	(0.000)	(0.000)	(0.000)	(0.003)	(0.029)
0.4,1	30	30	29.997	29.964	29.649
	(0.000)	(0.000)	(0.003)	(0.009)	(0.026)
0.5, 1	30	30	29.998	29.948	29.531
	(0.000)	(0.000)	(0.001)	(0.009)	(0.030)
0.6, 1	30	30	29.996	29.938	29.428
	(0.000)	(0.000)	(0.020)	(0.011)	(0.033)
0.75, 1	30	30	29.994	29.879	29.102
	(0.000)	(0.000)	(0.003)	(0.015)	(0.041)
1.0, 1	30	30	29.977	29.718	28.085
	(0.000)	(0.000)	(0.007)	(0.023)	(0.058)
1.5, 1	30	29.997	29.675	28.140	23.348
	(0.000)	(0.002)	(0.024)	(0.057)	(0.095)
2.0, 1	30	29.747	27.134	21.894	14.416
	(0.000)	(0.021)	(0.067)	(0.099)	(0.101)
3.0, 1	29.663	16.556	9.003	5.678	3.593
	(0.017)	(0.009)	(0.067)	(0.046)	(0.029)
4.0, 1	13.812	4.685	2.825	2.051	1.537
	(0.040)	(0.021)	(0.015)	(0.013)	(0.013)

decreases. However, there is still evidence of a significant improvement of the AE chart across all levels of the parameters.

Table 5 shows the ARL and SERL of the AE control chart with a chosen $\alpha = 0.005$ over 10,000 replications. As before, the "look-back" parameter r is varied. The results match the previously established patterns.

Based on these results, we now summarize the performance of the proposed adaptive control chart relative to the EWMA control chart. In general, the AE control chart performs uniformly better than the traditional EWMA control chart that uses a constant smoothing factor. Since the inertial effect is somewhat represented by the value of r, for small shifts in the process mean, a small value of r performs quite well. When the shift in the process mean is large, say four standard deviations or more, the magnitude of the "look-back" window size does not have a major impact on the ARL.

Table 5
Mean (Standard Error) of run length of adaptive control chart ($\alpha = 0.005$, μ_0, $= 0$, $\sigma = 1$).

AE				
μ_1, σ	$r=2$	$r=3$	$r=5$	$r=10$
0.1, 1	324.475	391.276	407.213	412.134
	(1.789)	(1.611)	(1.543)	(1.514)
0.25, 1	252.288	340.766	374.808	414.223
	(1.765)	(1.759)	(1.693)	(1.493)
0.40, 1	212.652	251.758	327.054	372.257
	(1.649)	(1.763)	(1.790)	(1.704)
0.50, 1	198.505	261.307	287.905	323.269
	(1.589)	(1.755)	(1.797)	(1.799)
0.60, 1	182.304	154.892	279.689	293.928
	(1.467)	(1.315)	(1.774)	(1.784)
0.75, 1	115.847	124.433	169.515	257.053
	(1.052)	(1.111)	(1.432)	(1.749)
1.0, 1	61.462	83.221	103.843	132.061
	(0.611)	(0.822)	(1.014)	(1.233)
1.5, 1	19.029	30.643	31.348	38.968
	(0.188)	(0.316)	(0.322)	(0.388)
2.0, 1	7.572	7.426	10.799	11.773
	(0.069)	(0.068)	(0.096)	(0.106)
3.0, 1	2.497	2.657	2.739	3.111
	(0.019)	(0.021)	(0.022)	(0.026)
4.0, 1	1.257	1.247	1.365)	1.399
	(0.005)	(0.005)	(0.007)	(0.008)

Table 6
Mean and standard deviation of run length of adaptive control chart (in-control ARL = 100).

	μ							
	0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50
Mean	101.19	81.29	50.64	27.45	15.26	6.39	3.40	1.99
S.D.	3.06	2.65	1.63	1.11	0.59	0.25	0.12	0.06

	μ				
	3.00	3.50	4.00	5.00	6.00
Mean	1.51	1.18	1.05	1.006	1
S.D.	0.03	0.01	0.01	0.003	0

For the EWMA control chart with a constant weight, we note that the ARL (SERL) are 30(0.000), for small shifts in the process mean (say $\mu_1 = 0.1$, 0.25, 0.40, 0.50, 0.60) and small smoothing constants (say $w = 0.1$, 0.3). This means that the shifts were not detected, for each of the simulations, over the period of 30 new observations for the shifted process.

The simulation results for the performance of the proposed adaptive control chart are shown for various values of the "look-back" window parameter, r. In particular, Tables 1, 3, and 5 show the results on the out-of-control ARL and the standard error of run length for values of $r = 2$, 3, 5, and 10. Comparison of these results indicate the sensitivity of the parameter, r. In general, the following behavior is observed. As the value of the parameter r increases, the out-of-control ARL increases. This is likely due to the inertial effect of the observations from the in-control process for large values of r. Further, the sensitivity of the parameter r based on the degree of shift (μ_1) of the process mean is also observed from Tables 1, 3, and 5, respectively. Finally, the chosen in-control ARL may also have an effect on the sensitivity of the parameter r.

Consider, for example, Table 1, for an in-control ARL of 20, and a quite small shift in the process mean as indicated by $\mu_1 = 0.1$. Observe the increase in the out-of-control ARL as r increases from 2 to 10. For the extreme values of r of 2 and 10, the increase is about 18.3%. For a moderate shift in the process mean as indicated by $\mu_1 = 1.0$, between values of $r = 2$ and 10, the increase in out-of-control ARL is about 50%. Finally, for large shifts in the process mean as indicated by $\mu_1 = 3.0$, the increase in out-of-control ARL,

Table 7
Mean and standard deviation of run length of adaptive control chart (in-control ARL = 500).

	μ							
	0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50
Mean	500.57	365.97	193.52	109.445	55.72	18.08	7.20	3.78
S.D.	21.2	15.93	8.65	5.16	2.49	0.81	0.29	0.14

	μ				
	3.00	3.50	4.00	5.00	6.00
Mean	2.004	1.51	1.24	1.03	1.002
S.D.	0.06	0.03	0.02	0.008	0.002

between $r = 2$ and 10, is about 19.8%. The greatest impact of r is found for shifts in the mid-range.

Similar inferences may be drawn from the results in Tables 3 and 5, respectively. Let us consider Table 3, for an in-control ARL of 100. Now, for a quite small shift in the process mean ($\mu_1 = 0.1$), as r increases from 2 to 10, the out-of-control ARL, increases by about 2.38%. For a moderate shift in the process mean ($\mu_1 = 1.0$), the increase in out-of-control ARL as r increases from 2 to 10, is about 20.2%. When the process mean shifts to $\mu_1 = 1.5$, the increase in out-of-control ARL, as r increases from 2 to 10, is about 40.6%. For a shift of the process mean to $\mu_1 = 2.0$, the corresponding increase in out-of-control ARL is about 54%. Finally, for large shifts in the process mean ($\mu_1 = 3.0$), as r increases from 2 to 10, the out-of-control ARL increases by about 32.7%. As the process mean shifts more ($\mu_1 = 4.0$), from $r = 2$ to 10, the out-of-control ARL increases by about 11.3%. It seems that a behavior, similar to that observed previously from Table 1 is observed.

Let us now consider Table 5, where the in-control ARL is 200. Now, for a small shift ($\mu_1 = 0.1$), as r changes from 2 to 10, the out-of-control ARL increases by about 27%. For a small shift of magnitude represented by $\mu_1 = 0.25$, the corresponding increase in out-of-control ARL, as r increases from 2 to 10, is about 64.2%. For a moderate shift in the process mean ($\mu_1 = 1.0$), the increase in out-of-control ARL is about 114.9%. For another moderate shift ($\mu_1 = 1.5$), as r increases from 2 to 10, the out-of-control ARL increases by about 104.8%. Finally, for large shifts in the process mean ($\mu_1 = 3.0$), the increase in out-of-control ARL is about 24.6%, as r increases from 2 to 10. These results demonstrate the sensitivity of the parameter r.

Let us study the sensitivity of the proposed chart, with respect to the parameter r, based on the chosen in-control ARL. Comparing the results in Tables 1, 3, and 5 as the in-control ARL changes from 20 to 100 to 200, we observe the following trend. As in-control ARL changes from 20 to 200, the increase in out-of-control ARL, as the parameter r increases from 2 to 10, is prone to greater changes as the in-control ARL increases. Such behavior is observed, in general, across all levels of the shift in the process mean. As an example, for a small shift ($\mu_1 = 0.5$), as r changes from 2 to 10,

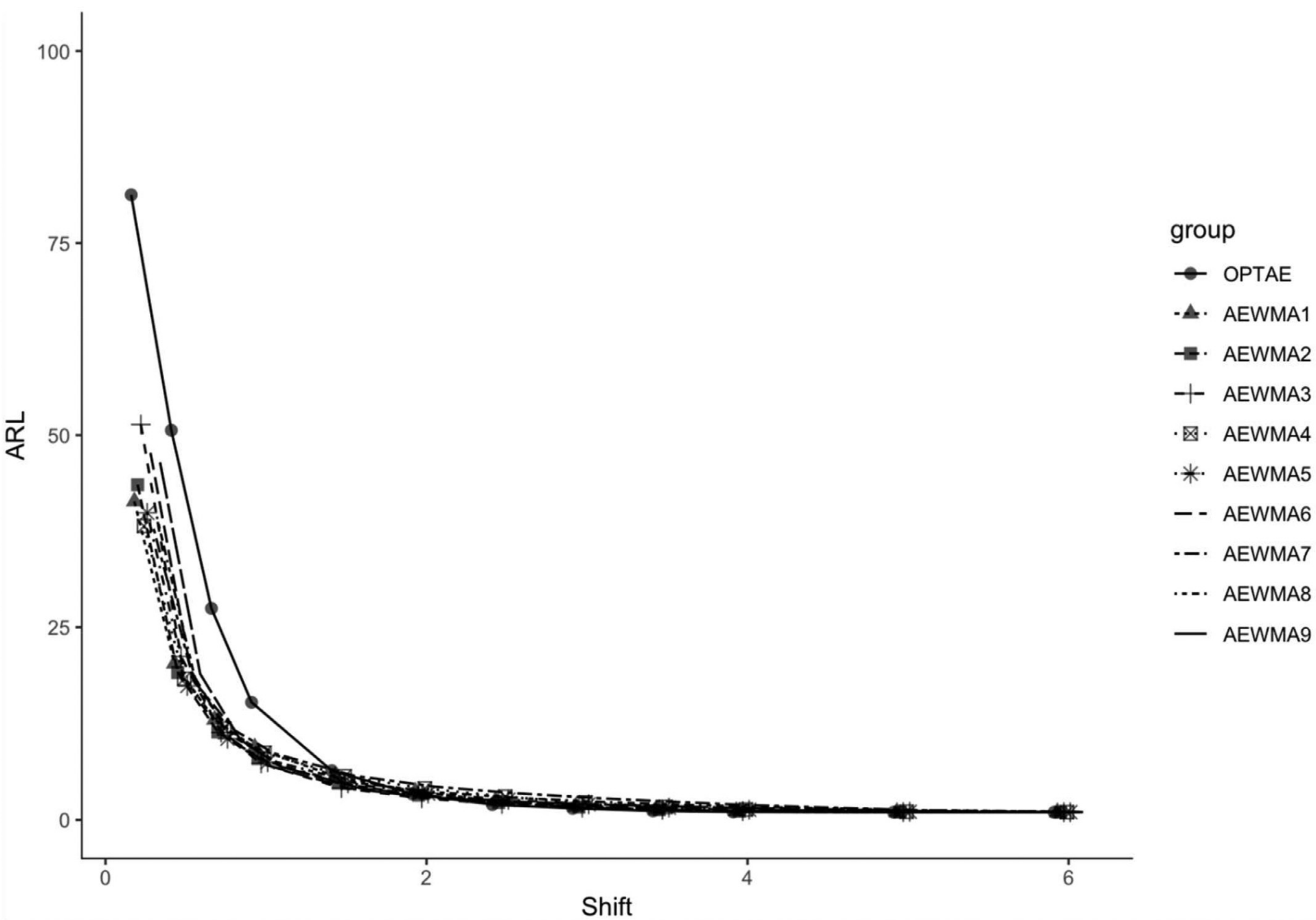

Fig. 1. Out-of-control ARL of proposed adaptive control chart versus Capizzi and Masarotto (2003) schemes (In-control ARL = 100)
Note: OPTAE is the proposed adaptive chart; others are Capizzi and Masarotto (2003) schemes using Huber's score function: AWEMA1 is $\mu_1 = 0.25$ and $\mu_2 = 4$; AWEMA2 is $\mu_1 = 0.50$ and $\mu_2 = 4$; AWEMA3 is $\mu_1 = 1.00$ and $\mu_2 = 4$; AWEMA4 is $\mu_1 = 0.25$ and $\mu_2 = 5$; AWEMA5 is $\mu_1 = 0.50$ and $\mu_2 = 5$; AWEMA6 is $\mu_1 = 1.00$ and $\mu_2 = 5$; AWEMA7 is $\mu_1 = 0.25$ and $\mu_2 = 6$; AWEMA8 is $\mu_1 = 0.50$ and $\mu_2 = 6$; AWEMA9 is $\mu_1 = 1.00$ and $\mu_2 = 6$.

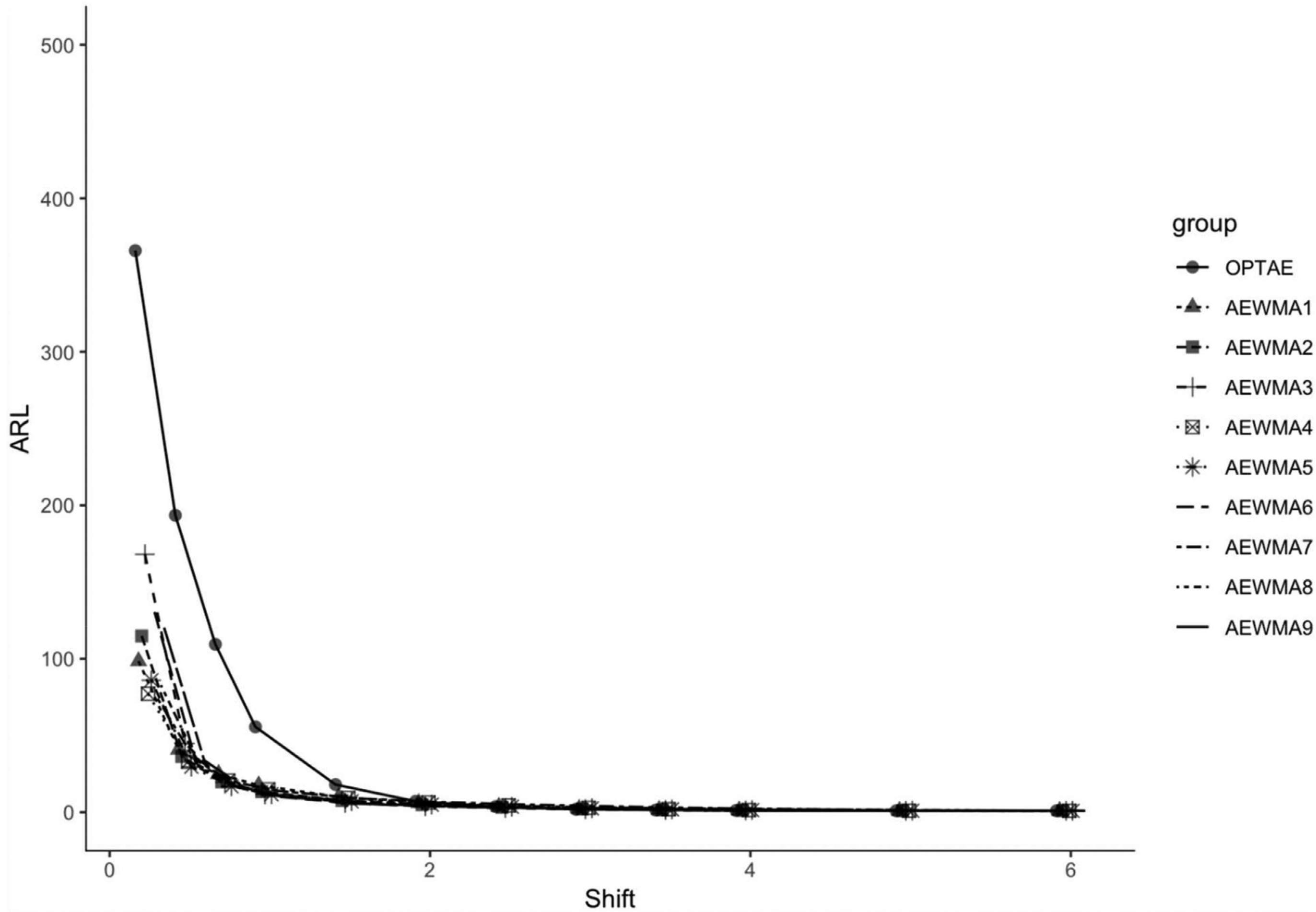

Fig. 2. Out-of-control ARL of proposed adaptive control chart versus Capizzi and Masarotto (2003) schemes (In-control ARL=500)
Note: OPTAE is the proposed adaptive chart; others are Capizzi and Masarotto (2003) schemes using Huber's score function: AWEMA1 is $\mu_1 = 0.25$ and $\mu_2 = 4$; AWEMA2 is $\mu_1 = 0.50$ and $\mu_2 = 4$; AWEMA3 is $\mu_1 = 1.00$ and $\mu_2 = 4$; AWEMA4 is $\mu_1 = 0.25$ and $\mu_2 = 5$; AWEMA5 is $\mu_1 = 0.50$ and $\mu_2 = 5$; AWEMA6 is $\mu_1 = 1.00$ and $\mu_2 = 5$; AWEMA7 is $\mu_1 = 0.25$ and $\mu_2 = 6$; AWEMA8 is $\mu_1 = 0.50$ and $\mu_2 = 6$; AWEMA9 is $\mu_1 = 1.00$ and $\mu_2 = 6$.

the difference in the out-of-control ARL increase is about 8.7%, as the in-control ARL changes from 20 to 200. For a moderate shift ($\mu_1 = 1.0$), the difference in the out-of-control ARL increase, under similar conditions, when in-control ARL changes from 20 to 200, is about 64.9%. For large shifts, the difference in out-of-control ARL, as r changes from 2 to 10, is smaller as in-control ARL increases from 20 to 200.

Since the proposed adaptive control chart was motivated by the AEWMA chart of Capizzi and Masarotto (2003), a comparison of their performance is of interest. This is studied here. The parameters to create these tables have been selected to match those in the Capizzi and Masarotto (2003) paper – i.e., Tables 6 and 7, in that paper. Note that Table 6 corresponds to an in-control ARL of 100 and Table 7 corresponds to an in-control ARL of 500, for different levels of the process mean shift (μ_1). The value of the parameter *r* is 100 in Table 6 and 500 in Table 7, respectively. In the Capizzi and Masarotto (2003) paper, the Huber's score function indicated by $\phi_{hu}(\bullet)$, was shown to perform better, in general. Hence, in our study, we compare the performance of our proposed adaptive control chart with the AEWMA chart based on $\phi_{hu}(\bullet)$. We report both the out-of-control ARL as well as the standard deviation of the run length. The Capizzi and Masarotto (2003) paper reports only the ARL. For the sake of brevity, the results from Capizzi and Masarotto (2003) Tables 6 and 7 are not reproduced here.

From Table 6 (in-control ARL = 100), we observe that for small shifts in the process mean ($\mu \leq 1.50$), the out of control ARL of the AEWMA chart based on Huber's function is shorter than that of the proposed adaptive control chart. However, the difference decreases rapidly with the degree of increase in the process mean. When the shift in the process mean is moderate to large ($\mu \geq 2.00$), the proposed adaptive control chart performs better than that of Capizzi and Masarotto. Obviously, as may be expected, for very large shifts ($\mu \geq 5.00$), the detection is usually on the very first subgroup drawn after the shift.

Fig. 1 shows the out-of-control ARL for different values of the shift of the process mean (μ_1) when the in-control ARL = 100. The proposed adaptive chart is labeled as OPTAE. The other schemes are those based on the choices of μ_1 and μ_2 for the $\phi_{hu}(\bullet)$ (Huber function) as shown in Table 6 of Capizzi and Masarotto (2003). As discussed earlier, the proposed adaptive chart dominates the AEWMA chart of Capizzi and Masarotto for the range of $\mu \geq 2.00$. The labels AEWMA1 to AEWMA9 correspond to nine choices of μ_1 and μ_2 combinations, respectively, for the Capizzi and Masarotto scheme based on Huber's score function. For example, AEWMA1 corresponds to $\mu_1 = 0.25$, $\mu_2 = 4$, and AEWMA9 corresponds to $\mu_1 = 1.00$, $\mu_2 = 6$, respectively.

Along similar lines, Table 7 demonstrates the performance of the proposed adaptive control chart, for chosen values of the mean shift as in Capizzi and Masarotto (2003), when the in-control ARL is selected to be 500 (Table 7 in Capizzi and Masarotto (2003)). Here again, we compare the performance of the proposed adaptive control chart with the Capizzi and Masarotto (2003) scheme

Table 8
Mean (Standard Error) of RUN Length of AE Chart and EWMA chart ($\alpha = 0.01$) for data from a chi-squared distribution.

AE						EWMA				
μ_0,σ_0	μ_1,σ_1	$r=2$	$r=3$	$r=5$	$r=10$	$w=0.1$	$w=0.3$	$w=0.5$	$w=0.7$	$w=0.99$
2, 4	3, 6	29.037	28.894	29.038	29.779	29.321	29.651	29.707	29.827	29.879
		(0.131)	(0.139)	(0.132)	(0.064)	(0.099)	(0.077)	(0.073)	(0.057)	(0.049)
2, 4	4, 8	27.021	27.237	28.211	28.182	28.132	28.276	28.618	28.125	28.291
		(0.091)	(0.182)	(0.111)	(0.192)	(0.091)	(0.071)	(0.067)	(0.076)	(0.079)
2, 4	5, 10	24.122	24.124	27.139	27.113	27.115	27.265	27.556	27.567	27.572
		(0.103)	(0.192)	(0.103)	(0.157)	(0.115)	(0.095)	(0.086)	(0.074)	(0.081)
2, 4	7, 14	19.455	20.546	23.134	23.557	24.427	24.528	24.528	24.902	24.229
		(0.093)	(0.122)	(0.111)	(0.174)	(0.117)	(0.081)	(0.099)	(0.082)	(0.102)

using Huber's score function. On comparison, we find that the proposed adaptive control chart dominates the Capizzi and Masarotto (2003) AEWMA chart for mean shifts in the range $\mu \geq 3.00$. As before, shifts of magnitude of six or more standard deviations are usually detected on the first subgroup drawn after the shift. Fig. 2 shows the ARL for the proposed adaptive chart and the Capizzi and Masarotto (2003) AEWMA charts for different levels of mean shifts.

In conclusion, since the degree of "small shift" (μ_1) and degree of "large shift" (μ_2) are most likely to be not known in practice, the utility of the proposed adaptive control chart is justified. In fact, a true comparison is not possible since our adaptive chart does not utilize a value of μ_1 and μ_2, which the Capizzi and Masarotto scheme does.

We compare the results from our Table 6 (in-control ARL = 100) and Table 7 (in-control ARL = 500) to those of the Capizzi and Masarotto (2003) scheme (Table 6 (in-control ARL = 100), and Table 7 (in-control ARL = 200)). As discussed previously, the proposed adaptive control chart dominates the AEWMA chart for moderate to large shifts in the process mean. Since the AEWMA chart requires the knowledge of μ_1 and μ_2, the degree of small and large shifts, which are not known in practice, we compare the performance of our adaptive control chart with the "best" among all of the 9 charts from Capizzi and Masarotto (2003), for a given shift (μ). For example, for an in-control ARL = 100, the percentage gains in ARL are 3.96%, 5.63%, 9.92%, 7.89%, and 14.02%, respectively, when the corresponding values of the shifted process mean (μ) are 2.50, 3.00, 3.50, 4.00, and 5.00, respectively. When the in-control ARL = 500, corresponding gains in ARL of our proposed adaptive control chart, relative to the "best" AEWMA Capizzi and Masarotto scheme, are 1.28%, 2.58%, 1.59%, and 0.96%, respectively, when corresponding values of the process mean (μ) are 3.00, 3.50, 4.00, and 5.00, respectively. It seems that the reduction in the out-of-control ARL, by using our proposed adaptive control chart, is higher for smaller values of the in-control ARL.

5. Conclusions

An adaptive EWMA-type control chart (AE chart) is proposed in the context of monitoring possible shifts in the process mean. One of the existing adaptive control charts in the literature, the AEWMA chart, does not seem to be readily implementable in practice for a couple of reasons. First, it requires a specification or an assumption about two kinds of possible shifts in the process mean to be a "small" and a "large" shift. One usually does not know the degree of shift that has taken place in the process in practice so this may be questionable. Second, the use of the AEWMA chart requires the solution of a complex two-step optimization problem in order to determine the chart parameters. These practical drawbacks lead to the motivation of our proposed adaptive control chart.

In the proposed AE chart, no quantitative assessment of the degree of a "small" or "large" shift is necessary. The chosen adaptive weights are influenced by the observations in the current period and those within the framework of the "look-back" window. A two-step computation, as in the case of the AEWMA chart, is also not necessary, but one does need to specify the look-back parameter, or use the entire past, which offers some nice practical flexibility to the user and gives the chart a more data driven look.

The proposed AE chart is seen to outperform the traditional EWMA chart in all of the comparisons. This is indeed noteworthy. Of course, the traditional EWMA chart uses a constant smoothing factor. In comparison with the AEWMA chart, it is easier and more practical to use. Also, the proposed AE chart is observed to outperform the AEWMA chart for certain ranges in the shift of the process mean, typically larger shifts. Another reason for the choice of the proposed AE control chart is that it is more in-control robust. For non-normal distributions, it has been shown that the AEWMA chart is non-robust (Zheng & Chakraborti, 2016). To explore the robustness of the proposed AE control chart, we performed a simulation study where observations were generated from a chi-squared distribution. For an in-control process, the observations were generated using a mean of $\mu_0 = 2$ and a standard deviation of $\sigma_0 = 4$. Note that, for a chi-squared distribution, the variance is related to the mean. For the shifted distributions, the following parameter combinations of (μ_1, σ_1) are used: (3, 6), (4, 8), (5, 10), and (7, 14), respectively. For a chosen $\alpha = 0.01$, Table 8 shows the ARL (SERL) of the AE control chart, with the "look-back" parameter r being 2, 3, 5, and 10, and also the OOC ARL (SERL) values of the EWMA control chart with the constant smoothing factor w being equal to 0.1, 0.3, 0.5, 0.7, and 0.99, respectively, using 1000 simulations for each parameter combination. It is observed from Table 8, based on OOC ARLs, the dominance of the AE control chart over the EWMA control chart.

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Declarations of interest

None.

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